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APPLICATION OF QUEUING THEORY TO CRICKET TEST MATCH

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ABSTRACT

In a cricket test match, two openers open the innings. That is, one pair is batting and the next batsman is waiting in a dressing room (i.e. waiting in a queue). We can derive an arrival rate and service rate from observation. So, we can apply the queuing theory to a cricket test match. As a result, we conclude that the result of the cricket test match is either a win (by any one of the two teams) or a draw with respective probabilities.

KEYWORDS: Cricket Test Match, Queue, Queuing Theory, M/M/1 Queuing Model

HISTORY OF A TEST CRICKET

Test cricket is played between international cricket teams who are full members of the International Cricket Council (ICC). Test matches consist of two innings per team, having no limit in their number of overs. The duration of tests has varied through test history, ranging from three days to timeless matches. Currently, the duration of tests is limited to five days. The earliest match now recognised as a test was played between England and Australia in March 1877, since then there have been over 2000 tests played by 10 teams. The frequency of tests has steadily increased because of the increase in the number of Test-playing countries.

INTRODUCTION

At the start of the cricket test match, two opening batsmen i.e. one pair of batsmen, open the innings. This pair of batsmen is considered as a customer being served by a single server i.e. cricket pitch. The one down batsman i.e. next batsman, is waiting in a dressing room i.e. this batsman is considered as a waiting customer in a queue.

From the innings of this two batsmen, we can derive the arrival rate and the service rate. We assume that, the service rate is slightly higher than the arrival rate. That means that, the time for which one pair is batting, is the service time for one pair of batsmen. And the time at which the incoming batsman occupies the pitch, is the arrival time for incoming batsman (i.e. the arrival time for a waiting batsman or customer). So, the service rate is slightly higher than the arrival rate. For this reason, we consider the utilization factor as 0.99. We observe that, the length of the queue is 1 customer. We also observe that, the average number of customers in the system is 2 customers (i.e. 1 pair is playing and 1 cricketer is waiting in a dressing room). We consider the following assumptions:

ASSUMPTIONS

- We assume that, at the start of the 1^{st} innings the one down batsman has already padded up and is waiting for his turn.
- It is assumed that, no batsman is retired hurt or retired absent in the test match (or in each innings).
- It is assumed that, there is no rain interruption during the test match.

- We assume that, the cricket stadium is a queuing system having components: arrival of cricketers, service mechanism, waiting of cricketers and departures.
- This queuing system is assumed to be in a steady-state.

We observe that, only one pair is served by a cricket pitch at a time. Other batsmen are waiting in a queue. Thus, the queuing model that best illustrates this cricketing model, is $M/M/1$ queuing model.

$M/M/1$ QUEUING MODELS

There are two models for the single server case $(c = 1)$. The first model has no limit on the maximum number in the system and the second model has finite limit on the maximum number. Both models assume an infinite-capacity source. Arrivals occur at the rate λ customers per unit time and the service rate is μ customers per unit time.

Our cricket model is the second model that has finite limit on the maximum number and an infinite-capacity source. In this model, the maximum number of customers is 40.

Letting $\rho = \frac{\lambda}{\mu}$, the probability P_n of having n customers in the system is

$$P_n = \rho^n P_0, n = 0,1,2,....$$

We find the probability P_0 of having 0 customers in the system, using the identity

$$P_0(1 + \rho + \rho^2 + \cdots) = 1.$$

Assuming, $\rho < 1$, the geometric series $1 + \rho + \rho^2 + \cdots$ will have the finite sum $\left(\frac{1}{1-\rho}\right)$. Thus, $P_0 = 1 - \rho$, provided $\rho < 1$.

Thus, the general formula for P_n is given by

$$P_n = (1 - \rho)\rho^n, n = 0,1,2,.... (\rho < 1).$$

The mathematical derivation of P_n imposes the condition $\rho < 1$ or $\lambda < \mu$. If $\lambda \geq \mu$, the geometric series will not converge and the steady-state probabilities will not exist. Because unless the service rate is larger than the arrival rate, queue length will continually increase and no steady-state can be reached.

STEADY-STATE MEASURES OF PERFORMANCE

The most commonly used measures of performance in a queuing situation are

$L_s =$ Expected number of customers in system

$L_q =$ Expected number of customers in queue

$W_s =$ Expected waiting time in system

$W_q =$ Expected waiting time in queue

$\bar{c} =$ Expected number of busy servers

The relationship between L_s and W_s (also L_q and W_q) is known as Little's formula. It is given as

$$L_s = \lambda W_s$$

$$L_q = \lambda W_q.$$

It describes that, as the waiting time in system increases (or decreases), expected number of customers in system increases (or decreases) and as the waiting time in queue increases (or decreases), expected number of customers in queue increases (or decreases) and vice-versa. This is exactly the case in cricketing model.

CALCULATION OF PROBABILITIES

As we know the maximum number of customers (batsmen) in this model is 40 and there is no limit in the number of over they play, the test match will have a result (a win by any one of the two teams), if more than 39 customers are served. That means, if 40 customers are served, the test match will have a result. In the context of cricketing model, this means that, if 40 wickets are fallen, the test match will have a result. Also, the test match will be drawn, if less than or equal to 39 customers are served. This means that, if less than or equal to 39 wickets are fallen, the test match will be drawn.

Thus, the probability of the test match resulting in a draw = P (less than or equal to 39 customers in the system) $= \sum_{n=0}^{39} P_n$

$=\sum_{n=0}^{39}(1-\rho)\rho^n$

$=0.331028$

The remaining probability will be of the test match resulting in a win or loss. That is, the probability of the test match resulting a win by any one of the two teams = P (more than 39 customers in the system) = $1-\sum_{n=0}^{39} P_n$

$=1-\sum_{n=0}^{39}(1-\rho)\rho^n$

$=1-0.331028$

$=0.668972$

In other words, we can say that, $33.10\ \%$ test matches result in a draw. And $66.89\ \%$ test matches result in a win (by any one of the two teams). That means that, out of 100 test matches 33.10 test matches result in a draw and 66.89 test matches result in a win (by any one of the two teams).

ACTUAL OBSERVATION

In some situation, we have observed that, there are only 30 or 34 or 35 wickets had fallen during a cricket test match, still the test match had resulted in a win or loss (a win by one team and a loss by other team). In this case, the

probabilities are varied but they are nearer to the probabilities, we have already obtained by calculation using queuing theory

In other situation, the team having won the toss or batting first, sets a big total, in their 1st innings. The other team in their 1st innings, gets following on by a captain of the opposite team. And starts their 2nd innings immediately, after their 1st innings and got out inside the lead of the opposite team. In this case, the number of wickets that would be falling is nearer to 30 or 30, still the team having a big total, wins a cricket test match. For this case also, we get the nearer probabilities.

In some other situation, the team having batted first, post a reasonable total around 250 or 300 in their 1st innings. The other team in their 1st innings, sets a big total, having reasonable lead. The opposite team in their 2nd innings got out inside this lead. So, the team having a big total, wins a cricket test match. In this case also, the number of wickets that would be falling is nearer to 30 or 30. Again, we get the nearer probabilities.

Thus, in general, we can say that, the probability of drawn test matches is 0.331028 and the probability of resulted test matches is 0.668972. These probabilities can be verified from the actual test records as follows:

Actual Test Records (As On December 2012)

Table 1

Teams	Matches	Won	Lost	Tied	Drawn	Drawn %
England	930	331	268	0	331	35.59
Australia	748	351	195	2	200	26.74
West Indies	488	158	162	1	167	34.22
India	468	115	149	1	203	43.38
New Zealand	377	72	154	0	151	40.05
Pakistan	370	115	101	0	154	41.62
South Africa	372	132	126	0	114	30.65
Sri Lanka	219	65	79	0	75	34.24
Zimbabwe	87	9	52	0	26	29.88
Bangladesh	75	3	65	0	7	9.33
Total	**4134**	**1351**	**1351**	**4**	**1428**	**34.54**

- Excluding the ICC World XI test against Australia.
- Including tied test matches.

From this table, we observe that, $34.54\ \%$ of test matches have been drawn, which is almost equal to $33.10\ \%$. We see that, there is no much difference between these percentages. Thus, we can verify our results.

Table 2

Total Matches Played	Number of Wins (By Any One of the Two Teams)	Number of Matches Drawn	Actual Probability of Drawn Matches
2067	1351	714	$\frac{714}{2067} = 0.34542$

CONCLUSIONS

As a result of this research paper, we conclude that, the probability of a cricket test match, resulting in a draw is 0.331028. And the probability of a cricket test match, resulting in a win (by any one of the two teams) is 0.668972.

In other words, out of 100 cricket test matches, that would be played, almost 33 test matches result in a draw and almost 67 test matches result in a win (by any one of the two teams)

The difference between the actual probability 0.345428 and the calculated probability 0.331028 is 0.0144 which is negligible. So, we can say that, the calculated probability gives the idea of the actual probability of a drawn cricket test match. Thus, queuing theory can be applied to a cricket test match to calculate the required probability.

The probability of a cricket test match, resulting in a draw is 0.331028 which is approximately equal to 0.333333 i.e. it is approximately equal to $\frac{1}{3}$. The probability of a cricket test match, resulting in a win (by any one of the two teams) is 0.668972 which is approximately equal to 0.666666 i.e. it is approximately equal to $\frac{2}{3}$. That means, the sum of the probabilities of a drawn test match and a resulted test match is 1 i.e. $\frac{1}{3}+\frac{2}{3}=1$

We also conclude that, the utilization factor of the cricket pitch is $\rho = 0.99$. That is, the pitch is occupied by one pair of batsmen for $99\ \%$ of the total time of the test match. Further, the probability of zero customers in the system is $P_0 = 1 - 0.99 = 0.01 = \frac{1}{100}$. That means that, the pitch is not occupied by any pair of batsmen for $1\ \%$ of the total time of the test match.

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